Section (A) : Distance formula, section formula, Slope, Collinearity

A-1.(i) Prove that the points (2a, 4a), (2a, 6a) and $(2a + \sqrt{3}a, 5a)$ are the vertices of an equilateral tr whose side is 2a.

(ii) Find the points which trisect the line segment joining the points (0, 0) and (9, 12).

A-2. (i) In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (– 7, 9

(ii) In which ratio P(2a – 2, 4a – 6)) divides Q (2a – 3, 3a – 7) and R(2a, 6a – 4). [16JM1104

A-3. (i) Find the value of λ such that points P(1, 2), Q(– 2, 3) and R(λ + 1, λ) are not forming a triangle?

(ii) Find the ratio in which the line segment joining of the points (1, 2) and (– 2, 3) is divided by th line 3x + 4y = 7

(iii) Find the harmonic conjugate of the point R (5, 1) with respect to points P (2, 10) and Q (6, -2

Section (B) : Area of Triangle & polygon, different forms of straight lines

B-1. A and B are the points (3, 4) and (5, – 2) respectively. Find the co-ordinates of a point P such that PA = PB and the area of the triangle PAB = 10. **[15JM110552]**

B-2. Find the area of the quadrilateral with vertices as the points given in each of the following : **[15JM110553]**
(i) (0, 0), (4, 3), (6, 0), (0, 3) (ii) (0, 0), (a, 0), (a, b), (0, b)

B-3. Reduce $x + \sqrt{3}y + 4 = 0$ to the :
(i) Slope intercepts form and find its slope and y-intercept.
(ii) Intercepts form and find its intercepts on the axes.
(iii) Normal form and find values of P and α. **[15JM110554]**

B-4. Find the equation of the straight line that passes through the point A(– 5, – 4) and is such that the portion intercepted between the axes is divided by the point A in the ratio 1 : 2 (internally). **[16JM110446]**

B-5. The co-ordinates of the mid-points of the sides of a triangle ABC are D(2, 1), E(5, 3) and F(3, 7). Find the length and equation of its sides. **[15JM110555]**

B-6. Find the straight line cutting an intercept of one unit on negative x-axis and inclined at 45° (in anticlockwise direction) with positive direction of x-axis **[16JM110447]**

Section (C) : Parametric form, angle between lines, parallel & perpendicular lines

C-1. Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line $x - y = 2$. Find their equations and also find the area of triangle bounded by the three lines. **[15JM110556]**

C-2. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight lines of which one has equation $x - 7y + 5 = 0$, then find equation of other line **[15JM110557]**

C-3. Find the equation of a straight lines which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line $2x + 3y + 4 = 0$ **[15JM110558]**

C-4. Through the point P(4, 1) a line is drawn to meet the line $3x - y = 0$ at Q where $PQ = \frac{11}{2\sqrt{2}}$. Determine the equation of line. **[16JM110448]**

C-5. From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. Find co-ordinates of point Q.

Section (D) : Position of point, linear inequation, perpendicular distance, image & foot

D-1. Plot the region **[15JM110559]**
(i) $6x + 2y \geq 31$ (ii) $2x + 5y \leq 10$ (iii) $8x + 3y + 6 > 0$ (iv) $x > 2$

D-2. Solve the following system of linear inequation graphically **[15JM110560]**
(i) $2x + y - 5 \geq 0$; $x - 3y + 10 \leq 0$
(ii) $-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x, y \geq 0$.

D-3. Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line $y = 3x - 4$.

D-4. Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point A(4, 8) and reflected line passes through the point B (8, 12). Compute the slope of the mirror. [16JM110449]

D-5. Find the nearest point on the line $3x + 4y - 1 = 0$ from the origin.

D-6. Find the position of the origin with respect to the triangle whose sides are $x + 1 = 0$, $3x - 4y - 5 = 0$, $5x + 12y - 27 = 0$. [15JM110561]

D-7. Find the area of parallelogram whose two sides are $y = x + 3$, $2x - y + 1 = 0$ and remaining two sides are passing through (0, 0).

Section (E) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

E-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of [15JM110562]

(i) Centroid
(ii) Circumcentre
(iii) Incentre
(iv) Excentre opposite to vertex (5, 12)

E-2. Find the sum of coordinates of the orthocentre of the triangle whose sides are $x = 3$, $y = 4$ and $3x + 4y = 6$. [16JM110450]

E-3. Find equations of altitudes and the co-ordinates of the othocentre of the triangle whose sides are $3x - 2y = 6$, $3x + 4y + 12 = 0$ and $3x - 8y + 12 = 0$. [15JM110563]

E-4. A triangle has the lines $y = m_1x$ and $y = m_2x$ for two of its sides, where m_1, m_2 are the roots of the equation $x^2 + ax - 1 = 0$, then find the orthocentre of triangle. [16JM110451]

E-5. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1, 0), where 't' is the parameter. [15JM110564]

E-6. Show that equation of the locus of a point which moves so that difference of its distance from two given points (ae, 0) and (–ae, 0) is equal to 2a is $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2-1)} = 1$. [16JM110452]

E-7. The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6), then find the locus of third vertex of triangle. [15JM110565]

E-8. Find the locus of point of intersection of the lines $x \cos\alpha + y \sin\alpha = a$ and $x \sin\alpha - y \cos\alpha = b$, where α is a parameter. [16JM110453]

Section (F) : Angle Bisector, condition of concurrency, family of straight lines

F-1. Find equations of acute and obtuse angle bisectors of the angle between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$. Also comment in which bisector origin lies. [15JM110566]

F-2. Find the value of λ such that lines $x + 2y = 3$, $3x - y = 1$ and $\lambda x + y = 2$ can not form a triangle. [16JM110454]

F-3. Find the equation to the straight line passing through

(i) The point (3, 2) and the point of intersection of the lines $2x + 3y = 1$ and $3x - 4y = 6$.
(ii) The intersection of the lines $x + 2y + 3 = 0$ and $3x + 4y + 7 = 0$ and perpendicular to the straight line $y - x = 8$. [15JM110567]

F-4. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines $ax + by + c = 0$ and $\ell x + my + n = 0$. [16JM110455]

Section (G) : Pair of straight lines, Homogenization

G-1. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the n^{th} power of the other, then prove that $(ab^n)^{\frac{1}{n+1}} + (a^nb)^{\frac{1}{n+1}} + 2h = 0$. [15JM110568]

G-2. For what value of λ does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represent a pair of straight lines? Find their equations, point of intersection, acute angle between them and pair of angle bisector. [15JM110569]

G-3. (i) Find the integral values of 'h' for which $hx^2 - 5xy + 4hy^2 + x + 2y - 2 = 0$ represents two real straight lines.
(ii) If the pair of lines represented by equation $k(k - 3)x^2 + 16xy + (k + 1)y^2 = 0$ are perpendicular to each other, then find k. [15JM110570]

G-4_. Find the equation of the straight lines joining the origin to the points of intersection of the line $lx + my + n = 0$ and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity. [16JM110456]

G-5_. Find the condition that the diagonals of the parallelogram formed by the lines $ax + by + c = 0$; $ax + by + c' = 0$; $a'x + b'y + c = 0$, $a'x + b'y + c' = 0$ are at right angles. Also find the equation to the diagonals of the parallelogram.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Distance formula, section formula, Slope, Collinearity

A-1. If the points $(k, 2 - 2k)$, $(1 - k, 2k)$ and $(-k - 4, 6 - 2k)$ be collinear, the number of possible values of k are
(A) 4 (B) 2 (C) 1 (D) 3

A-2. Given a ΔABC with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals : [15JM110572]
(A) 1 (B) 2 (C) 3 (D) Infinite

A-3_. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.
Statement-1: MP, MB, MQ are in G.P.
Statement-2 AP, AB and AQ are in HP.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

Section (B) : Area of Triangle & polygon, different forms of straight lines

B-1. Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (– 2, 3), (4, – 3) [15JM110573]
(A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

B-2. A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is : [15JM110574]
(A) $x + 2y = 5$ (B) $x - y + 1 = 0$ (C) $x + y - 3 = 0$ (D) $2x + y - 4 = 0$

B-3. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is

(A) 133 (B) 190 (C) 233 (D) 105

B-4. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15°. The equation of the line in the new position is : [16JM110457]

(A) $x - \sqrt{3}\,y - 2 = 0$ (B) $x - 2y - 2 = 0$

(C) $\sqrt{3}\,x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

B-5. In a ΔABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation $x + 2y = 16$. If the mid point of BC is (5, 6), then the equation of BC is

(A) $2x + y = 16$ (B) $x + y = 11$ (C) $2x - y = 4$ (D) $x + y = 10$

B-6. A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is : [16JM110458]

(A) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ (B) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(C) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ (D) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

Section (C) : Parametric form, angle between lines, parallel & perpendicular lines

C-1. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is :

(A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$ [15JM110577]

C-2. Find the equation of a straight line which passes through the point of intersection of the straight lines $x + y - 5 = 0$ and $x - y + 3 = 0$ and perpendicular to the straight line intersecting x-axis at the point (–2, 0) and the y-axis at the point (0, –3). [15JM110578]

(A) $2x + 3y + 10 = 0$ (B) $2x - 3y + 10 = 0$ (C) $2x - 5y + 10 = 0$ (D) $2x + 5y + 10 = 0$

C-3. Two particles start from the point (2, – 1), one moving 2 units along the line $x + y = 1$ and the other 5 units along the line $x - 2y = 4$. If the particles move towards increasing y, then their new positions are

(A) $\left(2-\sqrt{2}, \sqrt{2}-1\right), \left(2\sqrt{5}+2, \sqrt{5}-1\right)$ (B) $\left(2\sqrt{5}+2, \sqrt{5}-1\right), \left(2\sqrt{2}, \sqrt{2}+1\right)$

(C) $\left(2+\sqrt{2}, \sqrt{2}+1\right), \left(2\sqrt{5}+2, \sqrt{5}+1\right)$ (D) none of these

C-4. Two straight lines $x + 2y = 2$ and $x + 2y = 6$ are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally [16JM110459]

(A) $3x + 6y + 8 = 0$ (B) $3x + 6y = 14$ (C) $3x + 6y + 14 = 0$ (D) $3x + 2y = 10$

Section (D) : Position of point, linear inequation, perpendicular distance, image & foot

D-1. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line, $a^2x + a\,by + 1 = 0$ $\forall$ $a \in R$, $b > 0$ are : [15JM110579]

(A) $b \in (2, 4)$ (B) $b \in (0, 2)$ (C) $b \in [0, 2]$ (D) $(2, \infty)$

D-2 The point $(a^2, a + 1)$ is a point in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin, then [16JM110460]

(A) $a \geq 1$ or $a \leq -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$

(C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$

D-3. The image of the point A (1, 2) by the line mirror $y = x$ is the point B and the image of B by the line mirror $y = 0$ is the point (α, β), then :

(A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$

D-4. The equations of the perpendicular bisector of the sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is (1, –2), then the equation of the line BC is : [16JM110461]

(A) $14x + 23y = 40$ (B) $14x - 23y = 40$ (C) $23x + 14y = 40$ (D) $23x - 14y = 40$

Section (E) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

E-1. The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin, then the co–ordinates of C are : [15JM110581]

(A) (2a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2}, 0\right)$ (D) none

E-2. A triangle ABC with vertices A (– 1, 0),B (– 2, 3/4) & C (– 3, – 7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be : [15JM110582]

(A) (– 3, – 2) (B) (1, 3) (C) (– 1, 2) (D) none of these

E-3. Find locus of centroid of ΔABC, if B(1, 1), C(4, 2) and A lies on the line y = x + 3.

(A) 3x + 3y + 1 = 0 (B) x + y = 3 (C) 3x – 3y + 1 = 0 (D) x – y = 3

E-4. The locus of the mid-point of the distance between the axes of the variable line x cos α + y sin α = p, where p is constant, is [15JM110583]

(A) $x^2 + y^2 = 4p^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{2}{p^2}$

E-5. A variable straight line passes through a fixed point (a, b) intersecting the co–ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is : [15JM110584]

(A) bx + ay – 3xy = 0 (B) bx + ay – 2xy = 0 (C) ax + by – 3xy = 0 (D) ax + by – 2xy = 0

E-6. Consider a triangle ABC, whose vertices are A(–2, 1), B(1, 3) and C(x, y). If C is a moving point such that area of ΔABC is constant, then locus of C is : [16JM110462]

(A) Straight line (B) Circle (C) Ray (D) Parabola

E-7. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of 'c' is :

(A) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(C) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Section (F) : Angle Bisector, condition of concurrency, family of straight lines

F-1. The equation of bisectors of two lines L_1 & L_2 are 2 x – 16 y – 5 = 0 and 64 x + 8 y + 35 = 0. If the line L_1 passes through (– 11, 4), the equation of acute angle bisector of L_1 & L_2 is :

(A) 2 x – 16 y – 5 = 0 (B) 64 x + 8 y + 35 = 0

(C) 2x + 16y + 5 = 0 (D) 2x + 16y – 5 = 0

F-2. The equation of the internal bisector of ∠BAC of ΔABC with vertices A(5, 2), B(2, 3) and C(6, 5) is [16JM110463]

(A) 2x + y + 12 = 0 (B) x + 2y – 12 = 0 (C) 2x + y – 12 = 0 (D) 2x – y – 12 = 0

F-3. Consider the family of lines $5x + 3y - 2 + \lambda_1(3x - y - 4) = 0$ and $x - y + 1 + \lambda_2(2x - y - 2) = 0$. Equation of a straight line that belong to both families is -

(A) 25x – 62y + 86 = 0 (B) 62x – 25y + 86 = 0

(C) 25x – 62y = 86 (D) 5x – 2y – 7 = 0

F-4. The equation of a line of the system $2x + y + 4 + \lambda(x - 2y - 3) = 0$ which is at a distance $\sqrt{10}$ units fr
point A(2, –3) is [16JM11046

(A) $3x + y + 1 = 0$ (B) $3x - y + 1 = 0$ (C) $y - 3x + 1 = 0$ (D) $y - 3x - 1 = 0$

F-5. The lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, are concurrent at the point :

(A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (B) (1, 3) (C) (3, 1) (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Section (G) : Pair of straight lines, Homogenization

G-1. If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope c
the other line, then a = [15JM110586

(A) 1 (B) 2 (C) 4 (D) 16

G-2. The combined equation of the bisectors of the angle between the lines represented by
$(x^2 + y^2)\sqrt{3} = 4xy$ is [15JM110587]

(A) $y^2 - x^2 = 0$ (B) $xy = 0$ (C) $x^2 + y^2 = 2xy$ (D) $\frac{x^2 - y^2}{\sqrt{3}} = \frac{xy}{2}$

G-3. The equation of second degree $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$ represents a pair of straight lines. The distance between them is

(A) 4 (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $2\sqrt{3}$

G-4. The straight lines joining the origin to the points of intersection of the line $2x + y = 1$ and curve $3x^2 + 4xy - 4x + 1 = 0$ include an angle : [15JM110588]

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$